Svar till valda uppgifter i Johnson & Wichern 1998

2.6 a) $\boldsymbol{A}$ är symmetrisk

2.7 a) Egenvärden $\lambda_1 = 10, \lambda_2 = 5$.

2.10

$$\boldsymbol{B}^{-1} = 333333 \begin{pmatrix} 4.002001 & -4.001 \\ -4.001 & 4 \end{pmatrix}$$

$$\boldsymbol{A}^{-1} = -1000000 \begin{pmatrix} 4.002 & -4.001 \\ -4.001 & 4 \end{pmatrix}$$

$\boldsymbol{A}^{-1} = -3\boldsymbol{B}^{-1}$.

2.11 $p = 1$: $|a_{11}| = a_{11}$. $p = 2$:

$$\begin{vmatrix} a_{11} & 0 \\ 0 & a_{22} \end{vmatrix} = a_{11}a_{22} - 0 \cdot 0 = a_{11}a_{22}$$

2.12 $\boldsymbol{A} = \boldsymbol{P\Lambda P}'$. $|\boldsymbol{A}| = |\boldsymbol{P}||\boldsymbol{\Lambda}||\boldsymbol{P}'| = |\boldsymbol{\Lambda}| = \lambda_1\lambda_2\cdots\lambda_p$.

2.17

2.20 $\lambda_1 = 1.382, \boldsymbol{e}_1' = (0.8507, -0.5257)$, $\lambda_2 = 3.618, \boldsymbol{e}_2' = (0.5257, 0.8507)$.

2.23

2.24 (a) $\boldsymbol{\Sigma}^{-1} = \begin{pmatrix} \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{9} & 0 \\ 0 & 0 & 1 \end{pmatrix}$

(b) $\lambda_1 = 4, \boldsymbol{e}_1' = (1,0,0), \lambda_2 = 9, \boldsymbol{e}_2' = (0,1,0), \lambda_3 = 1, \boldsymbol{e}_3' = (0,0,1)$.

(c) $\lambda_1 = 1/4, \boldsymbol{e}_1' = (1,0,0), \lambda_2 = 1/9, \boldsymbol{e}_2' = (0,1,0), \lambda_3 = 1, \boldsymbol{e}_3' = (0,0,1)$.

2.25

2.26

2.27 (a) $\mu_1 - 2\mu_2$, $\sigma_{11} + 4\sigma_{22} - 4\sigma_{12}$.

(b)

(c)

(d)

(e) $3\mu_1 - 4\mu_2$, $9\sigma_{11} + 16\sigma_{22}$.

2.32

2.34 $\boldsymbol{b}'\boldsymbol{b} = 21, \boldsymbol{d}'\boldsymbol{d} = 15, \boldsymbol{b}'\boldsymbol{d} = -13$. $(\boldsymbol{b}'\boldsymbol{d})^2 = 169 \leq 21 \cdot 15 = 315$.

2.36 $\lambda_1 = 7, \lambda_2 = 1$. Maximum 7, minimum 1.

3.1 (a) $\bar{\boldsymbol{x}} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$.

(b) $\boldsymbol{e}_1 = \begin{pmatrix} 4 \\ 0 \\ -4 \end{pmatrix}, \boldsymbol{e}_2 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$.

(c) $L_{\boldsymbol{e}_1} = \sqrt{32}, L_{\boldsymbol{e}_2} = \sqrt{2}$. Vinkeln θ mellan $\boldsymbol{e}_1$ och $\boldsymbol{e}_2$: $\cos\theta = -0.5$.

3.6 $\boldsymbol{d}_1' = (-3, 0, -3), \boldsymbol{d}_2' = (0, 1, -1), \boldsymbol{d}_3' = (-3, 1, 2)$. Eftersom $\boldsymbol{d}_1 + \boldsymbol{d}_2 = \boldsymbol{d}_3$ är matrisen inte av full rang.

3.10 a) $\boldsymbol{X}_c = \begin{pmatrix} -2 & -1 & -3 \\ 1 & 2 & 3 \\ -1 & 0 & -1 \\ 2 & -2 & 0 \\ 0 & 1 & 1 \end{pmatrix}$. Eftersom kolumn1+kolumn2=kolumn3 gäller med $\boldsymbol{a}' = (1, 1, -1)$ att $\boldsymbol{X}_c\boldsymbol{a} = \boldsymbol{0}$.

3.11 $\boldsymbol{S} = \begin{pmatrix} 14808 & 14213 \\ 14213 & 15538 \end{pmatrix}$. $\boldsymbol{R} = \begin{pmatrix} 1 & 0.9370 \\ 0.9370 & 1 \end{pmatrix}$. $\boldsymbol{D}^{1/2} = \begin{pmatrix} 121.6881 & 0 \\ 0 & 124.6515 \end{pmatrix}$. $\boldsymbol{D}^{-1/2} = \begin{pmatrix} 0.0082 & 0 \\ 0 & 0.0080 \end{pmatrix}$.

3.12

3.14 (a) $\boldsymbol{b}'\boldsymbol{x}_1 = (2, 3)\begin{pmatrix} 9 \\ 1 \end{pmatrix} = 21$. $\boldsymbol{b}'\boldsymbol{x}_2 = 19, \boldsymbol{b}'\boldsymbol{x}_3 = 8$.

Medelvärde=$\frac{21+19+8}{3} = 16$. Stickprovsvarians=$\frac{(21-16)^2+(19-16)^2+(8-16)^2}{2} = 49$.

$\boldsymbol{c}'\boldsymbol{x}_1 = (-1, 2)\begin{pmatrix} 9 \\ 1 \end{pmatrix} = -7$. $\boldsymbol{c}'\boldsymbol{x}_2 = 1, \boldsymbol{c}'\boldsymbol{x}_3 = 3$.

Medelvärde=$\frac{-7+1+3}{3} = -1$. Stickprovsvarians=$\frac{(-7+1)^2+(1+1)^2+(3+1)^2}{2} = 28$.

(b) Medelvärde av $\boldsymbol{b}'\boldsymbol{X}$: 16
Medelvärde av $\boldsymbol{c}'\boldsymbol{X}$: -1

Stickprovsvarians hos $\boldsymbol{b}'\boldsymbol{X}$: 49
Stickprovsvarians hos $\boldsymbol{c}'\boldsymbol{X}$: 28
Stickprovskovarians mellan $\boldsymbol{b}'\boldsymbol{X}$ och $\boldsymbol{c}'\boldsymbol{X}$: -28

3.15

$$\bar{\boldsymbol{x}} = \begin{pmatrix} 5 \\ 3 \\ 4 \end{pmatrix}, \quad \boldsymbol{S} = \begin{pmatrix} 13 & -2.5 & 1.5 \\ -2.5 & 1 & -1.5 \\ 1.5 & -1.5 & 3 \end{pmatrix}$$

Medelvärde av $\boldsymbol{b}'\boldsymbol{X}$: 12
Medelvärde av $\boldsymbol{c}'\boldsymbol{X}$: -1

Stickprovsvarians hos $\boldsymbol{b}'\boldsymbol{X}$: 12
Stickprovsvarians hos $\boldsymbol{c}'\boldsymbol{X}$: -1
Stickprovskovarians mellan $\boldsymbol{b}'\boldsymbol{X}$ och $\boldsymbol{c}'\boldsymbol{X}$: -3